OSCILLATION POWER ACROSS THE HR DIAGRAM : SENSITIVITY TO THE CONVECTION TREATMENT

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Abstract

Solar-like oscillations are stochastically excited by turbulent convection. In this work we investigate changes in the acoustic oscillation power spectrum of solar-type stars by varying the treatment of convection in the equilibrium structure and the properties of the stochastic excitation model. We consider different stellar models computed with the standard mixing-length description by Böhm-Vitense (1958) and with a generalized formulation of the mixing-length approach by Gough (1976, 1977). We calculate the acoustic power generated by the turbulent convection which is injected stochastically into the acoustic pulsation modes.

Large differences in the oscillation powers are obtained depending on the choice of the assumed convection formulation. We show that the high-quality data Eddington will provide, will allow us to distinguish between theoretical predictions of acoustic power spectra obtained with different convection models.

1. Introduction

The amplitudes of solar-like oscillations are defined by the balance between excitation and damping. The oscillations are excited stochastically by the acoustic noise generated by the turbulent motion of the convective elements. In Samadi & Goupil (2001, Paper I hereafter) a theoretical formulation for the acoustic power injected into solar-like oscillations is proposed and which supplements previous theories. We refer the reader to Samadi (2001) for a detailed summary and discussion on some recent unsolved problems.

The excitation process depends on the assumed turbulence spectrum, as discussed, for example, by Samadi et al. (2001b) and Samadi (2001). It also depends crucially on the convection model to compute the stratification of the convectively unstable layers in the equilibrium model. The amount of energy injected into the oscillations depends strongly on the velocity of the convective elements.

The main goal of this work is to asses changes in the oscillation power spectrum due to modifications of the convection treatment in the equilibrium model. We consider two different formulations: the classical description of mixing-length theory by Böhm-Vitense (1958) and a

nonlocal generalization of the mixing-length formulation by Gough (1976, 1977). Additionally we study the dependence of the oscillation power on the assumed turbulence spectrum in the excitation model for both convection formulations.

The theoretical formulation in Paper I involves two free parameters. These parameters are calibrated such as to reproduce for a solar model the observed acoustic power spectrum (Section 2). In Section 3 we compute oscillation power spectra for several stellar models using the two convection formulations.

We conclude that Eddington's performance will allow us to distinguish between the two treatments of convection and consequently that Eddington will provide further constraints on convection models.

2. Power injected into solar-like oscillations

2.1. Theory of the stochastic excitation

The acoustic power P injected into the oscillations is defined (*e.g.*, Goldreich et al. 1994) in terms of the mode damping rate η, the oscillation mean-square amplitude $\langle A^2 \rangle$, the mode inertia I and oscillation frequency ω as :

$$P(\omega) = \eta \, \langle A^2 \rangle \, I \, \omega^2 \, . \tag{1}$$

The mean-square amplitude is defined by both the excitation by turbulent convection and by the damping process. It can be written as

$$\langle A^2 \rangle \propto \eta^{-1} \int_0^M \mathrm{d}m \, \rho \, w^3 \, \Lambda^4 \left(\frac{\mathrm{d}\xi_{\mathrm{r}}}{\mathrm{d}r} \right)^2$$
$$\times \left\{ \mathcal{S}_{\mathrm{R}}(\omega,m) + \mathcal{R}^2(m) \, \mathcal{F}^2(\xi_{\mathrm{r}},m) \, \mathcal{S}_{\mathrm{S}}(\omega,m) \right\} \, , \tag{2}$$

where ξ_{r} is the radial displacement eigenfunction, ρ the density, Λ is the mixing length, w the vertical rms velocity of the convective elements; $\mathcal{F}^2(\xi_{\mathrm{r}},m)$ is a function which includes the second derivative of ξ_{r}, and $\mathcal{R}^2(m)$ describes the ratio of the excitation by the entropy fluctuations to that by the Reynolds fluctuations. The source functions $\mathcal{S}_{\mathrm{R}}(\omega,m)$ and $\mathcal{S}_{\mathrm{S}}(\omega,m)$ describe the contributions from the Reynolds and entropy fluctuations, respectively, arising from the smaller scales of the turbulent cascade. Detailed expressions for $\langle A^2 \rangle$, $\mathcal{S}_{\mathrm{R}}(\omega,m)$, $\mathcal{S}_{\mathrm{S}}(\omega,m)$, $\mathcal{R}^2$ and $\mathcal{F}^2$ are given in Paper I.

The source functions include the turbulent kinetic energy spectrum $E(k)$, and the turbulent spectrum of the entropy fluctuations $E_s(k)$ which can be related to $E(k)$

by a simple expression (*e.g.*, Samadi et al. 2001a, Paper II hereafter). Both source functions $\mathcal{S}_R(\omega, m)$ and $\mathcal{S}_S(\omega, m)$ are integrated first over all eddy wavenumbers k, followed by an integration over the stellar mass M to obtain the acoustic power P (see Eq.2).

In the present work, ρ and w are obtained from the two equilibrium models, computed with the aforementioned two convection formulations. The eigenfunctions ξ_r and eigenfrequencies ω are obtained from two different oscillation codes and the k-dependence of $E(k)$ is inferred from different observations of the solar granulation and from theory.

2.2. THE EQUILIBRIUM MODELS

We consider two sets of stellar models: the first set consists of complete models obtained with the CESAM evolutionary code in which the convective heat flux is computed according to the classical mixing-length theory by Böhm-Vitense (1958, C-MLT hereafter). The momentum flux (sometimes referred to as turbulent pressure) is neglected in this set of models. The eigenfunctions are obtained from the adiabatic FILOU pulsation code by Tran Minh & Leon (1995). The detailed input physics used in this set of models is described in Paper II.

The second set consists of envelope models computed in the manner of Balmforth (1992a) and Houdek et al. (1999). In these models convection is treated with the nonlocal mixing-length formulation by Gough (1976, G-MLT hereafter), which consistently includes the momentum flux (i.e., the rr-component of the Reynolds stress tensor). The eigenfunctions are obtained from the nonadiabatic pulsation code by Balmforth (1992a), which includes both the Lagrangian perturbations of the heat and momentum fluxes in the manner of Gough (1977).

For both model sets the mixing length $\Lambda = \alpha H_p$, where H_p is the local pressure scale height and α the mixing-length parameter, is calibrated first to a solar model to obtain the helioseismically determined depth of the convection zone of 0.287 solar radii (Christensen-Dalsgaard et al. 1991).

2.3. MODELS FOR STELLAR TURBULENCE

Several turbulent spectra $E(k)$ were discussed in Paper II: the "Nesis Kolmogorov Spectrum" (NKS hereafter) and the "Raised Kolmogorov Spectrum" (RKS hereafter) were suggested from observations of the solar granulation by Espagnet et al. (1993) and Nesis et al. (1993). Here we also consider the "Broad Kolmogorov Spectrum" (BKS hereafter) by Musielak et al. (1994). These spectra are depicted in Figure 1. For wavenumbers $k > k_0$, where k_0 is the smallest wavenumber of the classical Kolmogorov spectrum, all spectra follow the classical Kolmogorov scaling law, $k^{-5/3}$. Only in the low-wavenumber range, $k < k_0$, where kinetic energy is injected into the turbulent cas-
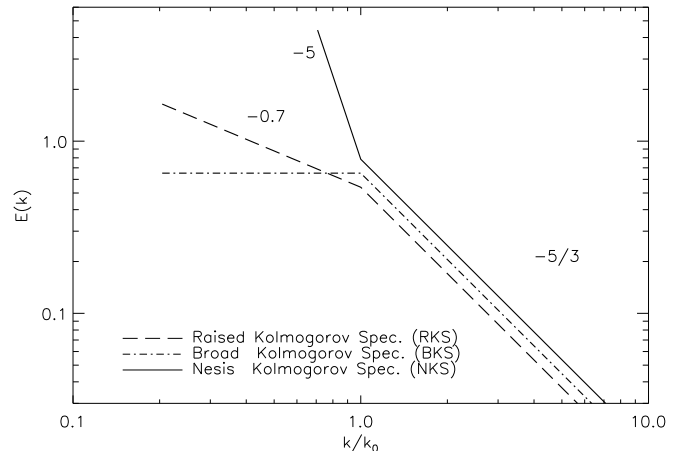

Figure 1. Kinetic turbulent spectra versus scaled wavenumber k/k_0.

cade, the spectra exhibit different scaling laws. These spectra were considered by Samadi et al. (2001b) to compute acoustic power spectra for various solar-type stars and are also considered in this paper.

From observations of the solar granulation Espagnet et al. (1993) and Nesis et al. (1993) suggest various values for k_0. Because of this ambiguity in the choice of k_0, we relate k_0 to the mixing length Λ by $k_0 = 2\pi / (\beta\Lambda)$, where β is a free parameter of order unity (Paper I). Another uncertainty in the formulation of the excitation model it the eddy correlation time scale. Because of our still poor understanding of turbulent convection the eddy correlation time scale is not well defined and therefore needs to be scaled, leading to an additional free parameter λ of order unity (*e.g.*, Balmforth 1992b). As it was demonstrated in Paper II the oscillation power computed for the Sun depends crucially on the values of the free parameters λ and β.

2.4. CALIBRATION OF THE FREE PARAMETERS

The mean square surface velocity v_s is related to the acoustic power $P(\omega)$ and the mode damping rate η by the expression :

$$v_s^2 = \xi_r^2(r_s) \, P(\omega) \, / \, 2\eta I \,, \tag{3}$$

where r_s is the radius at which the oscillations are observed.

The acoustic power P is computed for the two calibrated solar models using the aforementioned convection formulations. For the velocity estimates v_s, Eq.3, we assume the observed linewidths (damping rates) by Libbrecht (1988). The free parameters β and λ are calibrated for all turbulent spectra (see Section 2.3) and for both solar models to fit the estimated velocities v_s to the observations by Libbrecht (1988).

Figure 2 displays the results for the solar model computed with G-MLT. The corresponding calibrated values
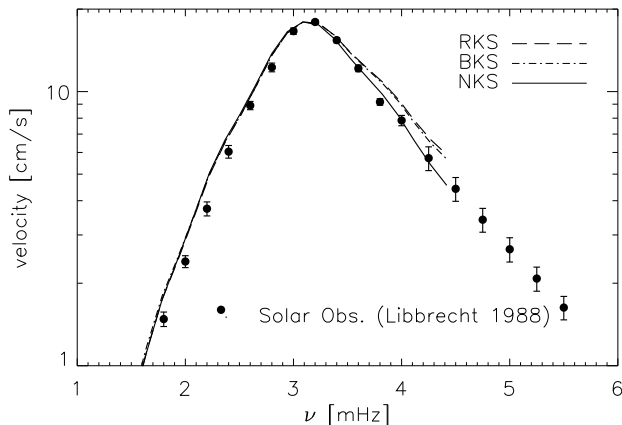
Figure 2. Computed surface velocities $v_{\rm s}$ (Eq.3) assuming various turbulent spectra (see Fig. 1) and G-MLT for computing the turbulent fluxes. The free parameters λ and β are calibrated to Libbrecht's (1988) velocity and linewidth (damping rate) measurements.

Table 1. Calibrated values of the parameters β and λ for the solar model computed with G-MLT.

spectrum	$\beta\lambda$	β	λ
RKS	0.6	1.96	0.31
BKS	1.6	4.06	0.39
NKS	2.6	3.11	0.83

of β and λ are listed in Table 1 (see Paper II for the calibration results of the models computed with the C-MLT).

At low frequencies the velocity $v_{\rm s}$ computed with G-MLT is in better agreement with the data than the results published in Paper II obtained with the C-MLT. At high frequencies the differences in $v_{\rm s}$ obtained with the various turbulent spectra are of similar small magnitude than the results of Paper II which assumed the C-MLT. Best agreement between theory and measurements is obtained with the NKS for both convection formulations.

3. SCANNING THE HR DIAGRAM

3.1. STELLAR MODELS

We consider various solar-type stars with masses between $1\,M_\odot$ and $2\,M_\odot$ in the vicinity of the main sequence. The model parameters are listed in Table 2, which correspond to the models considered previously by Samadi et al. (2001b). As for the solar models in Section 2 we consider two sets of stellar models: the first set, computed with the C-MLT, is adopted from Samadi et al. (2001b). The second set, computed with G-MLT, assumes the calibrated mixing length of the solar model discussed in Section 2. The acoustical cut-off frequencies are very similar between the two sets of models and are displayed in Table 2 for the first set.

Models	L	$T_{\rm eff}$	M	Age	ν_c
	$[L_\odot]$	$[{\rm K}]$	$[M_\odot]$	$[{\rm Gyr}]$	$[{\rm mHz}]$
A	12.1	6350	1.68	1.79	1.0
B	9.0	6050	1.44	3.05	1.0
C	6.6	6400	1.46	2.40	1.5
D	3.7	5740	1.08	7.33	1.5
E	3.5	6120	1.25	4.10	2.3
F	2.6	6420	1.25	1.76	3.6

Table 2. Model parameters of solar-type stars. The model age and acoustical cut-off frequency ν_c are listed for the models obtained with the CESAM evolutionary code which assumes the C-MLT for computing the convective heat flux.

3.2. DEPENDENCE ON CONVECTION MODELS

For all stellar models in both sets we assume the NKS and the calibrated values of β and λ quoted in Samadi et al. (2001b) and in Table 1.

The position in the HR diagram of all stellar models and a qualitative overview of their acoustic power spectra P are depicted in Fig. 3. Detailed results of P are shown in Fig. 5.

At high frequencies the differences in P between models computed with the C-MLT and G-MLT increase with increasing effective temperature $T_{\rm eff}$ and luminosity L. As discussed in Houdek (1996), the nonlocal formulation (G-MLT) predicts smaller temperature gradients in the upper superadiabatic region relative to the C-MLT. This means that convection is more efficient in models computed with G-MLT which leads to a different profile (depth-dependence) of the superadiabatic temperature gradient between the C-MLT and the G-MLT. The differences in the superadiabatic temperature gradient between the two model sets increase with L and $T_{\rm eff}$. These results are illustrated in Fig. 6 which shows the superadiabatic temperature gradient $\nabla - \nabla_{ad}$ versus $R_* - r$, with R_* being the radius of the star. With increasing L or $T_{\rm eff}$, the maximum of $\nabla - \nabla_{ad}$ is shifted more rapidly to deeper layers for the G-MLT models than for the C-MLT models. This leads to progressively larger differences in the convective velocities w and in the shape of the eigenfunctions between the two sets of models. These differences in w (note that P depends crucially on w, see Eq. 2) and in the shape of the eigenfunctions are particular large in the superficial layers and result in a larger amount of acoustic power injected into high frequency modes for models computed with the C-MLT.

There is an additional age effect: it can be seen from comparing the results of the models E and F (see Fig. 5). Model E has the same mass than model F but is older. The increase of the maximum acoustic power with age is found to be larger for models computed with the C-MLT than for the G-MLT models.
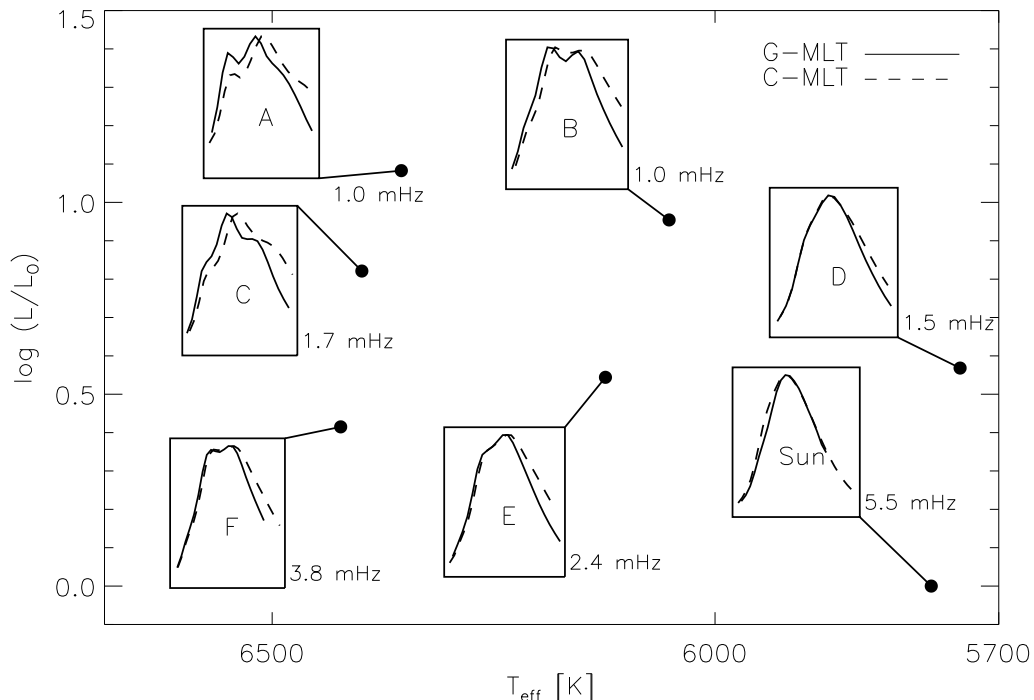
Figure 3. Positions of models in the HR diagram (filled circles). For each model of the two sets, the computed oscillation power spectra assume the NKS and are illustrated in an associated box : the solid curves correspond to models computed with G-MLT and the dashed curves display the results obtained with the C-MLT. The plot range of the abscissa starts from zero and extends to the acoustic cut off frequency, indicated for each model. The power spectra are normalized with their maximum values.

3.3. Dependence on turbulence models

We compute P for all the kinetic turbulent spectra discussed in Section 2.3. The results are shown in Fig. 4 for the models C and E. For the set of models computed with the C-MLT (top panels), the dependence of P on the assumed turbulence spectra is more pronounced for the hotter model C than for model E. This is not observed for the model set computed with G-MLT (bottom panels).

Models computed with the C-MLT exhibit a more pronounced decrease of the depth of the surface convection zone with increasing T_{eff}. This leads to a smaller extend of the excitation region for hotter models compared to the models computed with G-MLT. As discussed in more detail by Samadi et al. (2001b,c) a shallower excitation region results in a more pronounced frequency-dependence of P on the assumed turbulent spectra.

Model C, obtained with G-MLT, exhibit a deeper excitation region and consequently the dependence of P on the assumed turbulent spectra is smaller. This property can be explained by the larger efficacy with which G-MLT transports the convective heat flux.

3.4. Discussion

One of Eddington's tasks will be the continuous observation of stellar luminosity oscillations over a time period of 30 days with a frequency accuracy of $0.3\,\mu$Hz (Favata et al. 2000). Furthermore the large telescope of Eddington (approximately ten times larger than that of the COROT mission) will lead to a noise level of ~ 0.2 ppm for stars with a magnitude $m_v = 6$ and for an observing period of 30 days. For comparison, COROT will reach a noise level of 0.7 ppm (Auvergne & the COROT Team 2000) for stars with a magnitude $m_v \simeq 6$ and for a continuous observing period of 5 days.

Will the high-quality observations of the Eddington mission be accurate enough to allow us to distinguish between the changes in the predicted oscillation powers P of Section 3.2 and Section 3.3 ? In order to answer this question we estimate the expected accuracy of Eddington's measurements of P.

Kjeldsen & Bedding (1995) suggested a very simplified scaling law between oscillation amplitude ratios $(\delta L/L)/v_{\mathrm{s}}$ and T_{eff}:

$$\delta L/L \propto v_{\mathrm{s}}\, T_{\mathrm{eff}}^{-1/2}\,, \qquad (4)$$

assuming adiabatic oscillations in a purely radiative star. It suggests that the amplitude ratios scale inverse propor-
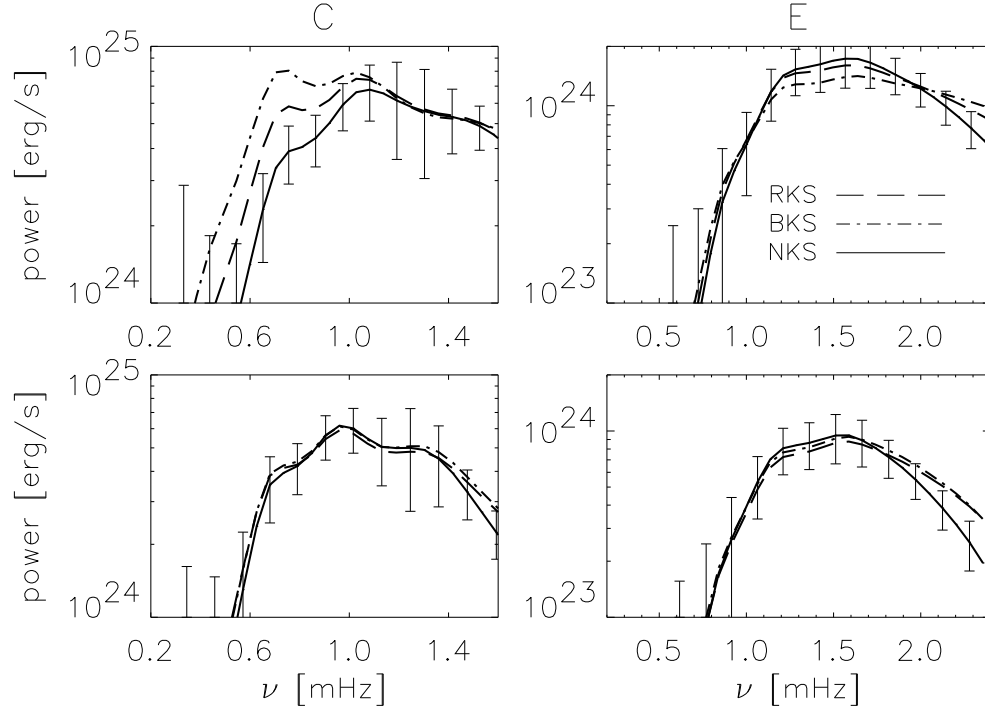
Figure 4. Oscillation power spectra computed for the models C and E assuming the NKS (solid curves), the RKS (dashed curves) and the BKS (dot-dashed curves). The top panel displays the results for models computed with the C-MLT and the bottom panel illustrates the results for models computed with G-MLT. Vertical error bars ΔP are obtained from Eq.(5) assuming an accuracy for $\eta/2\pi$ (resp. $\delta L/L$) of $0.3\,\mu Hz$ (resp. 0.2 ppm).

tionally to the effective temperature (but see also Houdek et al., 1999, Fig. 14, who found that the amplitude ratios scale directly with $T_{\rm eff}$).

According to Eq.(3) and Eq.(4) the relative error for P can be expressed as

$$\frac{\Delta P}{P} = 2\,\frac{\Delta\delta L}{\delta L} + \frac{\Delta\eta}{\eta}\,, \qquad (5)$$

where δL is obtained from Eqs.(3, 4). The damping rates η are supplied from the nonadiabatic pulsation code of Balmforth (1992a) assuming equilibrium models computed with G-MLT.

In Figure 4 and 5 the error bars ΔP are plotted according to Eq.(5) assuming a noise level of $\Delta(\delta L) \sim 0.2$ ppm and $\Delta(\eta/2\pi) \sim 0.3\,\mu Hz$.

We conclude that for hotter stars with a magnitude $m_v \leq 6$, Eddington will provide data of sufficient accuracy which will allow us to distinguish between the results obtained with the two considered convection formulations and consequently will provide further details on how to improve stellar convection models.

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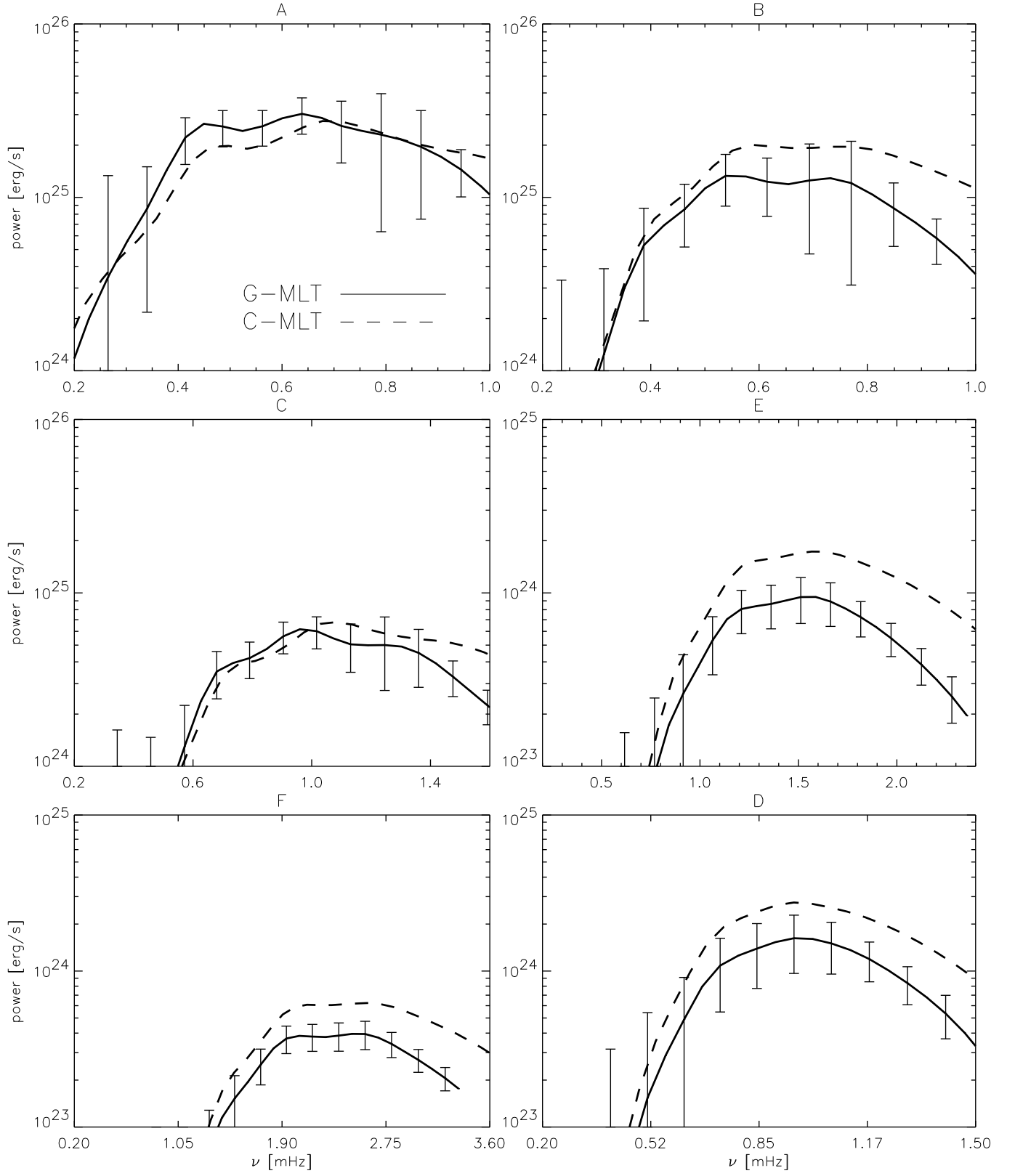Samadi R., 2001, In: SF2A-2001: Semaine de l'Astrophysique Francaise, E148

6

Figure 5. Oscillation power spectra computed for both sets of stellar models. The NKS turbulence model is assumed. The solid curves display the results for models computed with G-MLT and the dashed curves show the results for models computed with the C-MLT.
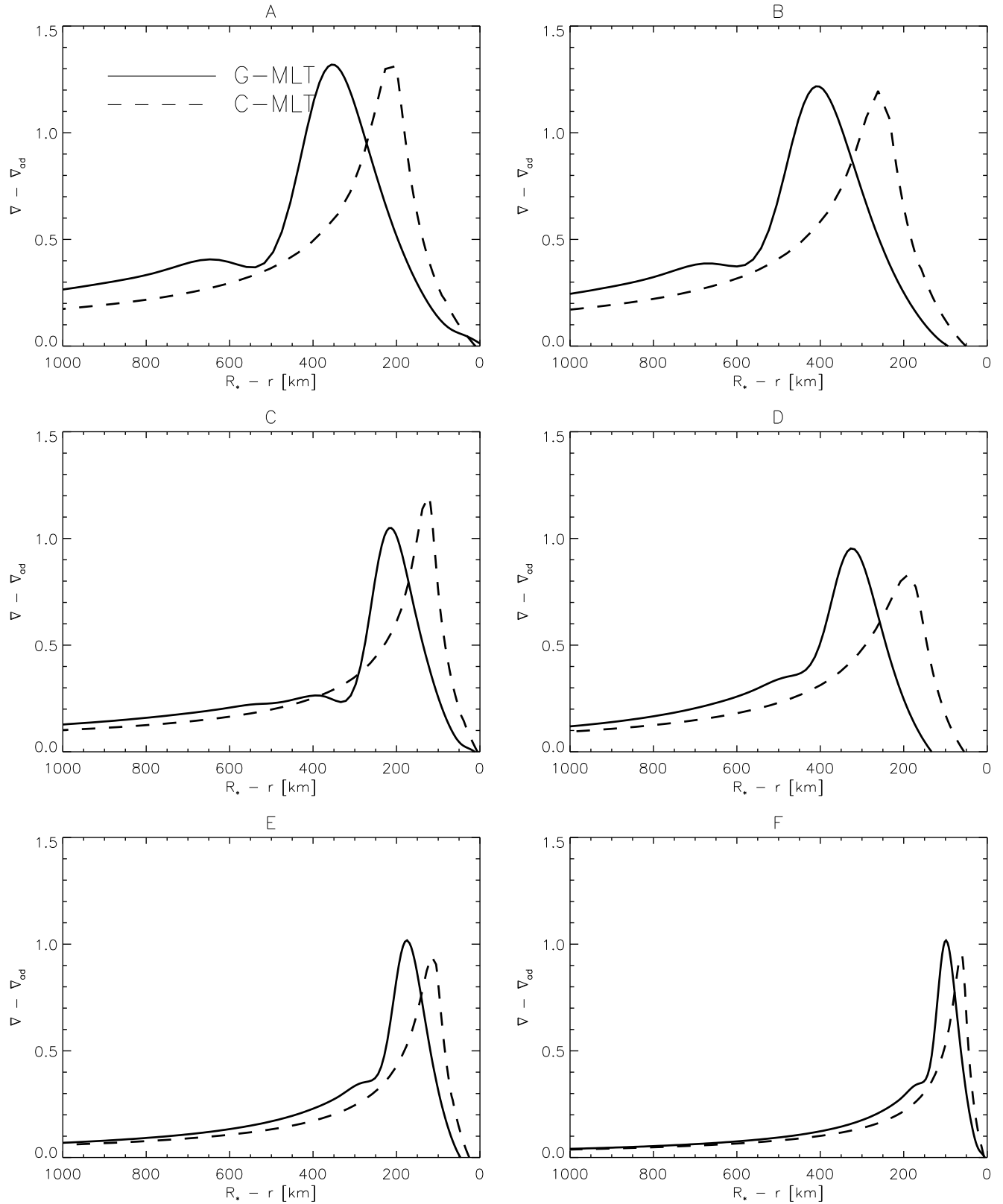
Figure 6. Depth dependence of the superadiabatic temperature gradient $\nabla - \nabla_{ad}$ for the two sets of stellar models with surface radius R_*. The solid curves display the results for models computed with G-MLT and the dashed curves are the results assuming the C-MLT.

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